**Probability:**

**Random Variables:**

Examples of random variables include:

Toss a fair coin n times

Often want to look at # of heads or tails

Range is 0 to n

The number of heads (or tails) is a variable

Roll a dice twice

Sum of the outcomes is also a RV

Have a set S = {1 … n}, and uniformly at random pick an integer m times

Number of distinct elements picked is a RV

A random variable, X, is a function from Ω to real numbers (R)

For the set S example:

Ω = <a1 … am> | ai is an element of {1 … n}

X(a1 … am) = number of distinct ints in the tuple

Suppose X is a random variable

We may want to know P(X = α) or P(X < α) or P(X > α)

Α is an element of R

Example:

Suppose we toss a fair coin n times, X = # of heads

To find P(X=7), look at E = set of tosses with exactly 7 heads

Two RVs, X and Y, are independent if, for every α and every β, X = α and Y = β are independent

The expectation of a RV, X, E[X] = Sum(P(X=α)\*α) for all α elements of R

Example:

Suppose we toss a fair coin, where X = 1 if outcome is H

E[X] = P(X=0)\*0 + P(X=1)\*1 = ½

The expected value of a fair dice is 3.5

Example:

Toss a fair coin n times, X = number of heads

E[X] = n/2 … (n+1)/2

E[X] = the the sum from 0 to n of (P(X=i)\*i)

P(X=i) = nCi \* 1/nn = n/2

Expectation is linear

IF X and Y are two RVs, E[aX + Y] = aE[X] + E[Y]

One way to find E[X] is to write X as the sum of a few other simple RVs

Let Y = number of heads in first n/2 tosses, and X = number in second n/2 tosses

X = Y + Z

3 CNF formula is an and of distinct or clauses, each of which is made up of exactly 3 literals

Given a 3 CNF formula, want to know if there is an assignment on which each clause is true (all (a1 … an) = T)

There are always 2n possible assignments

Given any CNF formula, the expected value is that 7/8 of the clauses will be satisfied